NPRE 446
Interaction of Radiation with Matter
Homework Assignments

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November 30, 2021

1 Homework 1

Attention:

1. Submit your homework pdf file to gradescope.com. The Entry Code is **YVKX3V**.
2. Explanation of the score: our brains typically consume about 0.2 Calories per minute. When actively thinking, our brains can kick it up to burning about 1 Calorie per minute. So instead of assigning each question with points, I will assign with Calories. For example, if a problem is given 10 Calories, it means you will need to burn about 10 Calories to solve the problem and the estimated time to solve the problem is about 10 minutes.
3. Because of the breath and depth of the content of the course, it is only possible to cover the essence during the lectures. One must read the relevant chapters in the textbooks to learn the details and gain deeper understandings.

Readings:

D. J. Griffiths and D. F. Schroeter, *Introduction to Quantum Mechanics*, 3rd edition, Cambridge University Press (2018).

- Section 1.2
- Section 1.4
- Section 1.6

1.1

What are the energies of an electron, a neutron, and a photon with wavelengths $\lambda = 1$ Å and 1 fm, respectively? (30 Calories)

1.2

What are the kinetic energy, velocity, and wavelength of thermal ($T = 300$ K, room temperature) neutrons? (15 Calories) Why are thermal neutrons useful for materials studies? Give at least three reasons. (5 Calories)

1.3

In Bohr's model of hydrogen, the electron in its ground state was supposed to travel in a circle of radius 5×10^{-11} m $= 0.5$ Å, held in orbit by the Coulomb attraction of the proton. According to classical electrodynamics, this electron should radiate, and hence spiral into the nucleus. Show that $v \ll c$ for most of the trip (so you can use the Larmor formula), and calculate the lifespan of Bohr's atom. (Assume each revolution is essentially circular.) [Griffiths's Electrodynamics: Page 487, Question 11.14] (30 Calories)

Notes: The Larmor formula of the radiation power of a moving charged particle with an acceleration a is:

$$P = \frac{\mu_0 q^2 a^2}{6\pi c}$$

$m_e = 9.11 \times 10^{-31}$ kg
$q_e = 1.6 \times 10^{-19}$ C
$\mu_0 = 4\pi \times 10^{-7}$ N/A^2
$g = 9.8$ m/s^2
$c = 3.0 \times 10^8$ m/s.

1.4

Griffiths and Schroeter, 3rd edition: Page 20, Problem 1.9. (30 Calories)

1.5

Griffiths and Schroeter, 3rd edition: Page 22, Problem 1.16. (30 Calories)

2 Homework 2

Readings:

D. J. Griffiths and D. F. Schroeter, *Introduction to Quantum Mechanics*, 3rd edition, Cambridge University Press (2018).

- Section 2.1
- Section 2.2

2.1

Consider a particle in a one dimensional infinite square well potential:

$$V(x) = \begin{cases} 0, & -a \leq x \leq a \\ \infty, & \text{otherwise} \end{cases}$$

where $a > 0$. Note that the range of the potential is different from the example in the lecture.

1. Compute the eigen energies and the eigen state wave functions. (20 Calories)
2. Compute $\langle x \rangle$, $\langle x^2 \rangle$, σ_x, $\langle p \rangle$, $\langle p^2 \rangle$, σ_p for the n-th eigen state. Compute the uncertainty relation quantity $\sigma_x \sigma_p$. Check whether the uncertainty relation is satisfied. Which state is closest to the uncertainty limit? (20 Calories)
3. If the initial wave function is

$$\Psi(x, 0) = \begin{cases} A(a + x), & -a \leq x \leq 0 \\ A(a - x), & 0 \leq x \leq a \\ 0, & \text{otherwise} \end{cases}$$

   for some constant A.

   (a) Sketch $\Psi(x, 0)$. Determine the constant A. (10 Calories)
   (b) Compute $\Psi(x, t)$. (20 Calories)
   (c) If we perform a measurement of the energy, what values may we get and with what probabilities? What's the expectation value? (10 Calories)

2.2

Griffiths and Schroeter, 3rd edition: Page 38, Problem 2.5. (30 Calories)

3 Homework 3

3.1

Griffiths and Schroeter, 3rd edition: Page 23, Problem 1.17. (30 Calories)

3.2

Griffiths and Schroeter, 3rd edition: Page 23, Problem 1.18. (30 Calories)

4 Homework 4

Readings:

S. Yip, *Nuclear Radiation Interactions*, World Scientific (2014).

- Section 3.1
- Section 3.2
- Section 3.3
- Chapter 4

4.1

1. What are isotopes, isobars, and isotones? Give one example of each of them. (10 Calories)
2. What are the common methods of isotope separation? Give at least two. (10 Calories)
3. Sketch the mass or charge distribution of a typical nucleus and explain the main features of the curve. (10 Calories)
4. What is binding energy $B(A, Z)$? (5 Calories)
5. Sketch the binding energy per nucleon B/A vs A and explain the main features of the curve. (15 Calories)
6. What is the Q value of a nuclear reaction? How do you compute the Q value from the binding energy? (10 Calories)

4.2

1. Compute the binding energy per nucleon $\frac{B}{A}$ for ^{4}He, ^{7}Li, ^{56}Fe, and ^{238}U using the semi-emperical formula. How do the results compare with those in the nuclear data? (20 Calories)
2. Compute the neutron separation energy S_n for ^{4}He, ^{7}Li, ^{56}Fe, and ^{238}U. (10 Calories)
3. Compute the proton separation energy S_p for ^{4}He, ^{7}Li, ^{56}Fe, and ^{238}U. (10 Calories)

5 Homework 5

Readings:

S. Yip, *Nuclear Radiation Interactions*, World Scientific (2014).

- Chapter 5. This chapter requires some more knowledge of quantum mechanics, which will be covered in 447/521. Therefore, we didn't exactly follow this chapter during the lecture. However, it is still beneficial if you can spend some time reading through, even if you cannot understand everything in this chapter.

5.1

1. In order to solve the 3-D time-independent Schrödinger equation for a 3-D isotropic potential $V(r)$, we use separation of variables. Show that the ordinary differential equation of the radial function $R(r) = \frac{u(r)}{r}$ is

$$-\frac{\hbar^2}{2m}\frac{d^2u(r)}{dr^2} + \left[V(r) + \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}\right]u(r) = Eu(r)$$

   (20 Calories)

2. Sketch the neutron-proton interaction $V(r)$. (10 Calories)

3. Solve and sketch the bound s-wave ($l = 0$) radial wave function for the deuteron. (20 Calories)

4. What is the binding energy of deuteron? How do you measure it experimentally? How does it compare with the average binding energy per nucleon B/A? (15 Calories)

5. What is the probability of finding the neutron or the proton outside the deuteron radius R_0? (15 Calories)

6. The radius of gyration is defined as $R_g = \langle r^2 \rangle^{\frac{1}{2}}$, where the angular bracket $\langle \cdot \rangle$ represents the expectation value in quantum mechanics. What is the R_g of the deuteron? (20 Calories)

6 Homework 6

Readings:

S. Yip, *Nuclear Radiation Interactions*, World Scientific (2014).

- Section 7.1 Concept of Scattering Cross Sections
- Section 8.1 Kinematics

6.1

1. What is the scattering differential cross section $\frac{d\sigma}{d\Omega}$? Explain the physical meaning. (20 Calories)
2. Using the neutron-proton interaction potential derived from the bound state of deuteron, what is the total neutron-proton scattering cross section we computed? What is the experimentally measured value of the total neutron-proton cross section? How do you explain the discrepancy? (10 Calories)
3. Why is water important for nuclear engineering? Give at least three reasons. (15 Calories)

6.2

Generally speaking, what are the common types of interaction of neutrons with matter? Explain each type. (15 Calories)

6.3

Derive the Q-equation:

$$Q = E_3\left(1+\frac{m_3}{m_4}\right) - E_1\left(1-\frac{m_1}{m_4}\right) - \frac{2}{m_4}\sqrt{m_1 m_3 E_1 E_3}\cos\theta$$

(20 Calories)

7 Homework 7

Readings:

S. Yip, *Nuclear Radiation Interactions*, World Scientific (2014).

- Section 9.2 Neutron Elastic Scattering
- Section 9.3 Energy and Angular Distribution

7.1

Consider a two-particle, say neutron with mass m and a target nucleus with mass Am, elastic scattering process. The energy of the incident neutron is E. The target nucleus is at rest before the collision.

1. Derive the relation between the energy E' and the outgoing angle θ of the scattered neutron in the Laboratory (L) coordination system. (10 Calories)
2. Show that in the Center-of-Mass (CM) coordination system the magnitude of the velocity each particle does not change before and after the collision. Only their directions change. (10 Calories)
3. Derive the relation between the energy E' and the outgoing angle θ_c of the scattered neutron in the Center-of-Mass (L) coordination system. (10 Calories)
4. Derive the relation between θ and θ_c:

$$\cos\theta = \frac{1 + A\cos\theta_c}{\sqrt{A^2 + 1 + 2A\cos\theta_c}}$$

   (10 Calories)

7.2

1. Sketch the typical energy distribution of scattered neutrons $F(E \to E')$. Explain its physical significance. (20 Calories)
2. What's the average energy loss of the neutrons. (10 Calories)

7.3

Sketch the energy-dependence of the total neutron scattering cross section $\sigma_s(E)$ for graphite and water. Explain the main features of the curves. (30 Calories)

8 Homework 8

Readings:

S. Yip, *Nuclear Radiation Interactions*, World Scientific (2014).

- Section 10.1 Compton Scattering
- Section 10.2 Photoelectric Effect
- Section 10.3 Pair Production
- Chapter 11 Charged Particle Stopping

8.1

What are the thermal neutron transmission coefficients of 1 mm thick light and heavy water respectively? (20 Calories)

8.2

1. Explain Compton scattering. (10 Calories)
2. Derive the Compton shift formula:

$$\Delta\lambda = \lambda' - \lambda = \lambda_c(1 - \cos\theta)$$

   where $\lambda_c = \frac{2\pi\hbar}{m_e c}$ is the Compton wavelength. (20 Calories)
3. Show that the kinetic energy of the recoil electron (Compton electron) is

$$T = \hbar\omega \frac{\alpha(1 - \cos\theta)}{1 + \alpha(1 - \cos\theta)}$$

   where $\alpha = \frac{\hbar\omega}{m_e c^2}$ is the ratio of the energy of the photon and the rest energy of the electron. (10 Calories)
4. Sketch the angular distribution of Compton scattering. Explain the main features of the curve. (10 Calories)

8.3

1. Explain photoelectric effect. (10 Calories)
2. Is an inner-shell electron or outer-shell electron more likely to be kicked out by a photon? Why? (10 Calories)
3. What is the fate of an excited atom produced by the photoelectric effect? (10 Calories)

8.4

1. How do the cross sections of each of the three kinds of interaction of photon with matter depend on Z (the atomic number of the materials) and $\hbar\omega$ (the energy of the photon). (10 Calories)
2. In the Z (the atomic number of the materials) vs $\hbar\omega$ (the energy of the photon) diagram, sketch which regions are dominated by which processes. (10 Calories)
3. Sketch the energy dependence of the attenuation coefficient of photons. Explain the main features of the curve. (10 Calories)

8.5

Sketch the incident particle energy dependence of the stopping power. Explain the main features of the curve. (10 Calories)

8.6

Sketch the Bragg curve of an α particle in air. Explain the main features of the curve. (10 Calories)