Interaction of Radiation with Matter

- Radiation (Ionizing): N, α/P, e, γ
  - Generation - radioactive decay
  - nuclear reaction

- Matter - Atom / Molecule (Atomic Physics)
  - Nucleus (Nuclear Physics)

- Interaction - depends on energy (EM, strong, weak)
§ Nuclear Physics

* Nuclear Properties

- Nuclear Physics - 1911 Rutherford's atomic model (atomic physics)
  - X scattering
  - discovery of nucleus

- Applications: 1. energy (most of our energy firms come directly/indirectly from NE)
  - (MeV)
  - 2. weapon
  - 3. medicine: diagnosis (X-ray, MRI), therapy
  - 4. materials research
  - 5. Carbon dating, food processing

- Theory: Quantum (mostly nonrelativistic, except β-decay), QM
  - Semi-phenomenological

- Terminology of Nuclides
  - mass number: number of nucleons (proton & neutron): \( A \) (unified atomic mass unit u)
  - \( A \) \( \equiv \) chemical symbol
  - \( Z \) \( \equiv \) number of protons
  - \( N \) \( \equiv \) number of neutrons
  - atomic number: number of protons
  - charge \( +Ze \)

- \( A \equiv Z (\sim 2Z) \)
  - \( A = Z + N \)
  - \( m_p \approx 2000 \text{ me} \) \( \Rightarrow M_{\text{atomic}} \approx M_{\text{nucleus}} \)

- \( X \) and \( Z \) are redundant

- Ex: \( ^1H, ^2H (D), ^3H (T) \)
  - \( ^3\text{He}, ^4\text{He} \)
  - \( ^{12}\text{C}, ^{16}\text{O} \) (half life 5700 years) carbon dating
  - \( ^{238}\text{U}, ^{235}\text{U} \)
"X-rays will prove to be a hoax."
"Heavier-than-air flying machines are impossible."
Lord Kelvin, President of the Royal Society, 1883.

✓ "It was a very poor and inefficient way of producing energy, and anyone who looked for a source of power in the transformation of the atoms was talking moonshine."
Ernest Rutherford, speech to the British Association for the Advancement of Science, 9/11/1933, reported in The Times newspaper on 9/12/1933. At the second day, 9/12/1933 Szilard proposed the concept of a nuclear chain reaction, which was shortly confirmed by Szilard and Fermi in 1939 (U.S. Patent 2,708,656).

✓ "There is not the slightest indication that nuclear energy will ever be obtainable. It would mean that the atom would have to be shattered at will."
Albert Einstein, 1932

✓ "Nuclear-powered vacuum cleaners will probably be a reality within ten years."
Alex Lewyt, president of Lewyt vacuum company, 1955

A rocket will never be able to leave the Earth's atmosphere.
The New York Times, January 13, 1920. The Times offered a retraction on July 17, 1969, as Apollo 11 was on its way to the moon.

"I think there is a world market for maybe five computers."
Thomas Watson, president of IBM, 1943

"There is no reason anyone would want a computer in their home."
Ken Olsen, founder of Digital Equipment Corporation, 1977

"Television won't be able to hold on to any market it captures after the first six months. People will soon get tired of staring at a plywood box every night."
Darryl Zanuck, executive at 20th Century Fox, 1946

"Almost all of the many predictions now being made about 1996 hinge on the Internet's continuing exponential growth. But I predict the Internet will soon go spectacularly supernova and in 1996 catastrophically collapse."
Robert Metcalfe, founder of 3Com, 1995

"Two years from now, spam will be solved."
"We will never make a 32 bit operating system."
"640K ought to be enough for anybody."
Bill Gates, founder of Microsoft, 2004

There's no chance that the iPhone is going to get any significant market share.
- same $Z$, different $A/N$, --- isotope $^3$H, $^3$He, $\beta^+$ decay
- same $A$, different $Z/N$ --- isobar
- same $N$, different $Z/A$ --- isotope $^2$H, $^3$He

\[ Z = N \quad \text{stable nuclei} \quad N \geq Z \]

Chart of Nuclides

- nuclear properties

static: size, mass, charge, binding energy, angular momentum (spin)
  parity, magnetic dipole, electric quadrupole, excited states

dynamic: decay, reaction, cross section,

\( J(L) - 3/2 \sim \beta^{-} \sim 5/2 \)
Size, Distribution of Charge and Mass

- Size of atoms: potential \( \frac{1}{r} \), size \( \to \infty \)
- Distance in solids \( \sim A^0 \)

- Size of nuclei: distribution
- Charge distribution \( \approx \) mass distribution

Measurement/Effect of the distribution

(1) Electron diffraction \( \lambda \approx 10 \text{ fm} \), \( E = 100 \text{ MeV} - 1 \text{ GeV} \)

- Elastic scattering
  \( E_i = E_f \), \( P_i = h\mathbf{k}_i = P_f = h\mathbf{k}_f \Rightarrow \mathbf{k}_i = \mathbf{k}_f = \frac{2\pi}{\lambda} \)

- Wave vector transfer
  \( k = 2\frac{2\pi}{\lambda} \sin \theta \)

- \( I(2\theta) \sim \frac{d\sigma}{d\xi} \sim S(k) \)
  - Structure factor

- Fermi's golden rule
  \[ F(k_i, k_f) = \int V(r) \phi_{k_i}(r) \phi_{k_f}^*(r) \, dr \]
  \[ F(k) = \int V(r) e^{-ik \cdot r} \, dr \]
  \[ V(r) = \frac{Ze^2}{4\pi\varepsilon_0} \int \frac{\phi(r') \, dr'}{|r - r'|} \]

- Form factor
  \[ F(k) = \int \phi(r) e^{-ik \cdot r} \, dr ' \]
  \[ S(k) = |F(k)|^2 \]

- \( S(k) \) spherical Bessel function
(2) Isotope shift (X-ray, optical)

- Electron can enter nucleus \( \Rightarrow \) size of nucleus affects electron energy

Ideally, \( V(r) = -\frac{Ze^2}{4\pi\varepsilon_0} \frac{1}{r} \) - point charge

In reality, \( V(r) = \begin{cases} -\frac{Ze^2}{4\pi\varepsilon_0} \frac{1}{r}, & r > R \\ -\frac{Ze^2}{4\pi\varepsilon_0 R} \left( \frac{3}{2} - \frac{1}{2} \left( \frac{r}{R} \right)^2 \right), & r < R \end{cases} \)

Consider the energy of an electron in both cases:

\[
\langle V \rangle = \int \Psi^* V \Psi \, dv \\
\langle V' \rangle = \int \Psi'^* V' \Psi' \, dv = \int \Psi'^* V \Psi \, dv + \int \Psi'^* V' \Psi' \, dv \\
\text{Assuming } \Psi \text{ does not change much}
\]

\[
\Delta E = \langle V' \rangle - \langle V \rangle = \int_{r < R} \Psi'^* (V' - V) \Psi \, dv
\]

For is \( \Delta E = \frac{e^2}{4\pi\varepsilon_0} \frac{4\pi A^4}{a_0^3} e^{-2\frac{2\pi r}{a_0}} \left\{ \frac{1}{r} - \frac{1}{R} \left( \frac{3}{2} - \frac{1}{2} \left( \frac{r}{R} \right)^2 \right) \right\}
\]

In principle, \( \Delta E \) can be determined from the difference between experiments and calculation. But \( \Delta E \approx 10^4 E \) (relativistic effect, other electrons)

- K\(\alpha\) X-ray isotope shift (2p \(\rightarrow\) 1s) \( \Delta E \sim 10^6 E \)

\[
E_{K\alpha}(A) - E_{K\alpha}(A') = \left[ E_{2p}(A) - E_{1s}(A) \right] - \left[ E_{2p}(A') - E_{1s}(A') \right]
\]

\( E_{2p}(A) \approx E_{2p}(A') \) because \( \Psi_{2p}(r=0) = 0 \)

\[
E_{K\alpha}(A) - E_{K\alpha}(A') \approx -E_{1s}(A) + E_{1s}(A') = -\Delta E(A) + \Delta E(A) = -\frac{2}{5} \frac{Ze^2}{4\pi\varepsilon_0 a_0^3} (A^3 - A'^3)
\]
\[ \Delta E \approx 10^{-7} E \]

- optical isotope shift, laser spectroscopy
- muonic atom: muon like electron, but \( m_\mu \approx 207 \) MeV, so \( a_{\mu} \approx \frac{a_e}{207} \)

muon is closer to nucleus, isotope shift is more evident

13) Coulomb energy of nuclei (mirror nuclei)

\[ ^3\text{H} \text{ and } ^3\text{He} : \text{same nuclear energy, different Coulomb energy due to repulsion} \]

\[ ^{13}\text{N}_6 \text{ and } ^{13}\text{C}_7, \quad ^{39}\text{Ca}_{19} \text{ and } ^{39}\text{K}_{20} \]

Only works for mirror nuclei, otherwise \( n \) & \( p \) are not in the same states

\[ E_C = \frac{3}{5} \left( \frac{Z^2 e^2}{4 \pi \varepsilon_0 K} \right) \]

\[ \Delta E_C = \frac{3}{5} \left( \frac{e^2}{4 \pi \varepsilon_0} \right) \left( \frac{1}{K} \right) \left( Z^2 - (Z-1)^2 \right) \approx \frac{3}{5} \left( \frac{e^2}{4 \pi \varepsilon_0} \right) \left( \frac{1}{K} \right) A \frac{A^2}{R_0} \]

- measurement
  1) \( \beta^+ \) decay, energy of positron
  2) nuclear reaction \( p + ^{11}\text{B} \rightarrow ^{14}\text{C} + n \)
(4) Rutherford Scattering

\[ E_x \]

\[ I \]

- Coulomb
- Rutherford
- Coulomb + strong

(5) $\alpha$ decay

\[ V \]

- Coulomb

\[ T_x \]

- 6 MeV

- $R$

- Charge distribution of $n$ & $p$. 

\[ dT^2 P(r) \]

\[ 0.5 \text{ fm} \]

\[ 1.5 \text{ fm} \]
Mass, Abundance

- mass spectrometer

\[ qE = qVB \]

- mass doublet method: measure the difference between two close-lying masses \( C_{H_2O}, C_{O_{18}} \)

- use mass spectrometer to measure abundance \( C_i \)

- average mass \( \bar{m} = \sum C_i m_i \) (in periodic table)

- isotope separation using mass spectrometer

- laser isotope separation

\[ \text{Laser 1} \quad \text{Laser 2} \]

acetate \( A_2 \) only
isotope shift
Binding Energy

\[ B(A,Z) = \left\{ Zm_p + Nm_n - [m(A,Z) - Zm_e] \right\} c^2 \]

\[ = \left[ Zm_H + Nm_n - m(A,Z) \right] c^2 \]

Atomic mass

Electronic binding energy \(\sim 10^{-eV} \)

Nuclear binding energy \(\sim 1 GeV \)

- Binding energy per nucleon

\[ c^2 = 931.50 \text{ MeV/amu} \]

Graph showing\( B/A \) vs.\( A \) with points for\( ^{56}\text{Fe} \),\( ^{238}\text{U} \),\( ^{6}\text{Li} \), and\( ^{4}\text{He} \). Arrows indicate fusion and fission processes.
- Q value of nuclear reaction

\[ i + I \rightarrow f + F + Q \]

\[ Q = \left[ (m_i + m_I) - (m_f + m_F) \right] c^2 \]

- \( Q > 0 \) : exothermic
- \( Q < 0 \) : endothermic, provided by the kinetic energy of \( i \)

- Energy conservation

\[ T_i + m_i c^2 + T_I + m_I c^2 = T_f + m_f c^2 + T_F + m_F c^2 \]

\[ Q = (T_f + T_F) - (T_i + T_I) \]

\[ Q = (B_f + B_F) - (B_i + B_I) \]
Separation energy: energy required to separate particle \( \alpha \)

\[
S_a = B(A, Z) - B(A-A', Z-Z')
= [m_a(A', Z') + m(A-A', Z-Z') - m(A, Z)]c^2
\]

- neutron separation energy:

\[
S_n = B(A, Z) - B(A-1, Z)
= [m_n + m(A-1, Z) - m(A, Z)]c^2
\]

- proton separation energy:

\[
S_p = B(A, Z) - B(A-1, Z-1)
= [m_p + m(A-1, Z-1) - m(A, Z)]c^2
\]

- n/p separation energy ~ ionization energy in atomic physics

- \( S_n \) (even N) > \( S_n \) (odd N) for a given Z
- \( S_p \) (even Z) > \( S_p \) (odd Z) for a given N

- Same nucleons tend to pair

\[\text{Mev} \begin{array}{c|c|c|c}
\text{N=120} & \text{118} & \text{119} & \text{Lead} \\
\hline
10 & & & \\
\end{array} \]

- Abundance (odd - even)

\[\begin{array}{c}
\text{Z} \\
\hline
\text{odd A:} \\
\text{stable isotope} \\
\end{array} \quad \begin{array}{c}
\text{Z} \\
\hline
\text{even A:} \\
>1 \text{ stable isotope} \\
\end{array} \]
Semiempirical mass formula,

\[ B = a_v A - a_s \frac{A^2}{A^3} - a_c \frac{Z(Z-1)}{A^2} - a_{\text{sym}} \frac{(N-Z)^2}{A} + \frac{g_p}{\sqrt{A}} \]

\[ \text{Liquid Drop Model} \]

\[ a_v \quad a_s \quad a_c \quad a_{\text{sym}} \quad a_p \]

15.835 \quad 18.33 \quad 0.714 \quad 23.20 \quad \begin{cases} 11.2 & \text{odd } N, \text{ even } Z \\ 11.2 & \text{odd } N, \text{ odd } Z \end{cases} \\
\begin{cases} 0 & \text{even } N, \text{ odd } Z \\ -11.2 & \text{even } N, \text{ even } Z \end{cases}

1) \( a_v \): If all nucleons attract each other, \( B \propto A(A-1) \).

\[ -\frac{B}{A} \propto \text{constant} \Rightarrow \text{nucleons only attract their neighbors} \]

2) \( a_s \): surface \( B \propto R^2 \propto A^2 \)

3) \( a_c \): Coulomb energy \( a_c = \frac{3}{5} \frac{e^2}{\pi \epsilon_0 R_0} \approx 0.7 \text{ MeV}, \ R_0 = 1.2 \text{ fm} \)

\[ \text{favors more } p \text{ than } n \]

\[ \Rightarrow \text{deviation from } N=Z \text{ line} \]

4) \( a_{\text{sym}} \): stable nuclei \( N \approx Z \)

5) \( a_p \): odd - even

mass parabola

\[ m(A, Z) = Z M_H + N M_n - \frac{B(A, Z)}{c^2} \]

parabola for a given \( A \)

\[ \frac{m}{A^2} = 0 \Rightarrow Z_{\text{min}} = A \left( \frac{1}{2} + \frac{1}{2} \left( \frac{a_{\text{sym}}}{a_v} \right) \right)^\frac{1}{2} \]

Small \( A \) : \( Z_{\text{min}} \approx A \)

Large \( A \) : \( Z_{\text{min}} \approx \frac{A}{2} < 0.4 \ A \)
- **Nuclear Angular Momentum/Spin**

Electronic angular momentum \( j \) is split into orbital \( l \) and spin \( s \), \( j = l + s \)

Nuclear angular momentum / nuclear spin \( I, m \)

\[ I^2 = \hbar^2 I(I+1), \quad I_z = m \hbar, \quad m = -I, \cdots, I \]

- Often, \( I \) is determined by one valence nucleon \( I = j \)

- Sometimes, \( I = j_1 + j_2 \)

- Sometimes, one odd nucleon and the core \( I = j_{core} + j \)

\[
I = \begin{cases} 
\text{half-integer}, & \text{odd } A \\
\text{integer}, & \text{even } A 
\end{cases}
\]

- **Nuclear parity** \( I^\pi \)

\[ \psi(-r) = \pm \psi(r) \]
Nuclear Electromagnetic Moments

1. \[ T_l = (\frac{(-1)^l}{(-1)^{l+1}}) \text{ for } n^{th} \text{ electric moment} \]
2. Magnetic moment \( \mu = \frac{e}{2m} \pi r^2 = \frac{e}{2m} mvr = \frac{e}{2m} L \)

\[ \mu = \frac{e\hbar}{2m} \]

- \(\mu_B \gg \mu_N\), electronic magnetism \(\gg\) nuclear magnetism

\[ \gamma = \frac{g\mu_N}{\hbar} \]

- \(g = 1\) for the orbital of proton
- \(g = 0\) for neutron
- \(g = \frac{1}{2}\) for spin of proton, neutron, electron

Free proton and neutron \(g = 5.5856912\) proton\(\quad g = -3.8260837\) neutron

\(\Rightarrow\) proton and neutron have internal structures

- Pairing of nucleons leads to zero angular momentum

3. Electric quadrupole moment

\[ eQ = e \int \psi^*(3\mathbf{z}^2 - r^2)\psi \, dv \]
Nuclear Force / Residual Strong Interaction

- Basic properties
  1. Short range: negligible at atomic size.
  2. Stronger than Coulomb at nuclear size, overcome Coulomb repulsion of protons.
  3. Electrons feel no nuclear interaction.
  5. Spin-dependent: odd-even effect.
  6. A repulsive term; keeps nucleons from collapse.
  7. Noncentral or tensor component: does not conserve orbital angular momentum, which is a constant of motion under central force.
Deuteron

Deuteron vs hydrogen atom
in nuclear physics vs in atomic physics

Just as the measured Balmer series of electromagnetic excitations between excited states of the electron led to the understanding of the structure of a hydrogen atom, in principle, transitions between the excited states of a deuteron can lead to the understanding of the nuclear structure. However, deuteron has no excited states. The only "excited" state is the unbound system.

- Binding energy
  \[ B(\text{H}) = \left[ m(\text{H}) + m(n) - m(\text{H}) \right] c^2 = 2.224 \text{ MeV} \]

1. Mass spectroscopy
   \( m(\text{H}) \)
   \( m(\text{H}) \)
   mass doublet method
   \( \text{C}_5 \text{H}_{12} \) & \( \text{C}_6 \text{D}_{12} \)
   \( \text{CsD}_{12} \) & \( \text{C}_6 \text{D}_{12} \)

2. \( \text{H} + n \rightarrow \text{H} + \gamma \)

3. \( \gamma + \text{H} \rightarrow \text{H} + n \), photo dissociation

Note: \( B(\text{H}/2) < 8 \text{ MeV} \), the average over weakly bound
The $r$ component of $\psi$, $\psi(r) = \frac{u(r)}{r}$

$$-\frac{\hbar^2}{2m} \frac{d^2 u}{dr^2} + V(r) u(r) = E u(r)$$

bound state solution $E < 0$

$r < R$ \[ u(r) = A \sin(k_1 r) + B \cos(k_1 r) \]

$r > R$ \[ u(r) = C e^{-k_2 r} + D e^{k_2 r} \]

$u(0) = 0 \Rightarrow B = 0$; otherwise $R(r) = \frac{u(r)}{r}$, $\int \frac{d}{dr} R(r) \, dr = \lambda$

$u(\infty) = 0 \Rightarrow D = 0$.

continuity is $\psi(r)$, $\frac{d\psi}{dr} |_{r=R}$ \Rightarrow \text{continuity of } u(r), \frac{d u}{dr} |_{r=R}$

\[ A \sin(k_1 R) = C e^{-k_2 R} \Rightarrow k_1 \cot (k_1 R) = -k_2 \]

\[ A R \cos(k_1 R) = -C k_2 e^{-k_2 R} \Rightarrow k_1 \cot (k_1 R) = -k_2 \]

$E = 2 \text{MeV}$, $R = 2.1 \text{fm} \Rightarrow V_0 = 35 \text{ MeV}$

- $2 \text{MeV} \ll 35 \text{ MeV}$, weak

- If $V_0 = 35 \text{ MeV}$ is a little weaker, deuteron wouldn't form.

- no fusion \Rightarrow no heavy nuclides \Rightarrow cosmological consequence

- small "turn over" in $u(r)$
spin and parity

\[ l = \text{even} \leftrightarrow \text{even parity} \]

\[ S_n = \pm \frac{1}{2}, \quad S_p = \pm \frac{1}{2} \]

two possibilities

\[ (1) \quad l = 0, \quad S_n = S_p = \frac{1}{2} \quad \text{parallel} \quad \text{: s state} \]

\[ (2) \quad l = 2, \quad S_n = -S_p = -\frac{1}{2} \quad \text{parallel} \quad \text{: d state} \]

magnetic dipole moment

\[ \mu = \mu_n + \mu_p = g_{S_n} \mu_N S_n + g_{S_p} \mu_N S_p = \frac{1}{2} \mu_N (g_{S_n} + g_{S_p}) \]

\[ = \frac{1}{2} \mu_N (-3.826084 + 5.585691) = 0.879804 \mu_N \]

very close to measurement 0.8574376

So a mixture of s state and d state

\[ |\psi\rangle = C_s |10\rangle + C_d |12\rangle \]

\[ \mu = C_s^2 \mu_s + C_d^2 \mu_d \]

\[ \mu_d = \frac{1}{4} (3 - g_{S_n} - g_{S_p}) \]

\[ \Rightarrow \quad C_s^2 = 0.96 \quad \quad C_d^2 = 0.04 \]

electric quadrupole moment \( Q \)

\( n \) and \( p \) have zero electric quadrupole moment.

\( Q \) of \( ^2\text{H} \) must come from \( l \) (\( l = 2 \))
**neutron-proton scattering**

- D has no excited states \( \Rightarrow \) Information of N-P interaction is limited.

\[
r < R \quad u(r) = A \sin(k_1 r) \quad k_1 = \frac{\sqrt{2m(E + V_0)}}{\hbar}
\]

\[
r > R \quad u(r) = C' \sin(k_2 r) + D' \cos(k_2 r) \quad k_2 = \frac{\sqrt{2mE}}{\hbar}
\]

\[
= C \sin(k_2 r + \delta)
\]

**Continuity**

\[
A \sin(k_1 R) = C \sin(k_2 R + \delta)
\]

**Smoothness**

\[
A k_1 \cos(k_1 R) = C k_2 \cos(k_2 R + \delta)
\]

\[
\Rightarrow k_1 \cot(k_1 R) = k_2 \cot(k_2 R + \delta)
\]

\( \delta \) is determined by \( V_0, R, \) and \( E \) phase shift.

**Attractive potential** \(-V_0\)

**Repulsive potential** \(V_0\)
\[ \psi_i = A \frac{\sin kr}{kr} = \frac{A}{2ikr} \left[ e^{i kr} - e^{-i kr} \right] \quad \text{s-wave approximation} \]

\[ \psi_0 = B \frac{\sin(kr + \delta_0)}{kr} = \frac{B}{2ikr} \left[ e^{i(kr + \delta_0)} - e^{-i(kr + \delta_0)} \right] \]

\[ = \frac{B e^{-i \delta_0}}{2ikr} \left[ e^{i(kr + 2 \delta_0)} - e^{-i kr} \right], \quad A = B e^{-i \delta_0} \]

\[ \psi_3 = \psi_0 - \psi_i = \frac{A}{2ikr} \left[ e^{i(kr + 2 \delta_0)} - e^{-i kr} \right] \]

\[ = \frac{A}{2ikr} (2i \delta_0) e^{i kr} \]

\[ J_s = \frac{\hbar k}{m} \frac{\sin^2 \delta_0}{(kr)^2} 1A_1^2 \]

\[ J_i = \frac{\hbar k}{m} 1A_1^2 \]

\[ d \sigma = \frac{J_3}{J_i} \implies \frac{d \sigma}{d \Omega} = \frac{\sin^2 \delta_0}{k^2} \]

\[ k = k_2 = \frac{\sqrt{2mE}}{\hbar} \approx 0.016 \text{ fm}^{-1}, \quad E \leq 10 \text{ keV} \]

\[ k_1 = \frac{\sqrt{2mU_0 + E}}{\hbar} \approx \frac{\sqrt{2mU_0}}{\hbar} = 0.92 \text{ fm}^{-1} \]

\[ k_1 \cot k_1 R = k_2 \cot (k_2 R + \delta_0) = -\alpha \]

\[ \alpha = k_1 \cot k_1 R = 0.2 \text{ fm}^{-1}, \quad R = 2 \text{ fm} \]

\[ \frac{\sin^2 \delta_0}{k_2^2} = \frac{\cos k_2 R + \frac{\alpha}{k_2} \sin k_2 R}{k_2^2 + \alpha^2} \approx \frac{1 + \frac{\alpha R}{k_2}}{\alpha^2}, \quad k_2 R \ll 1 \]

\[ \sigma = 4\pi \frac{\sin^2 \delta_0}{k_2^2} = 4\pi \frac{1 + \alpha R}{\alpha^2} = 4.6 \text{ b} \]

\[ 1 \text{ b (barn)} = 10^{-28} \text{ m}^2 = 10^{-24} \text{ cm}^2 \]
\[ S = S_0 + S_p = \begin{cases} 1 & \text{triplet} \\ 0 & \text{singlet} \end{cases} \]

\[ \sigma = \frac{3}{4} \sigma_t + \frac{1}{4} \sigma_s \]

\[ \sigma_t = 4.6 \text{ b} \quad \sigma_s = 67.8 \text{ b} \quad \sigma = 20.4 \text{ b} \]

orthohydrogen \quad H-H \quad S=1 \quad "parallel" \n
parahydrogen \quad H-H \quad S=0 \quad "anti-parallel" \n
- Scattering length \[ a = -\frac{\sin \delta_0}{k} \]

As \( k \to 0 \), \( \delta_0 \to 0 \), otherwise \( a \to \infty \), so \( a \sim -\frac{\delta_0}{k} \)

\[ \Psi_s = \frac{A}{z i k r} \left( e^{2iz \delta_0} - 1 \right) e^{ikr} \approx \frac{A}{k} \delta_0 \frac{e^{ikr}}{r} = -A \frac{e^{ikr}}{r} \]

- Sign of \( a \)

\[ u(r) = \sin k(r-a) \]

- Higher order when \( k \to 0 \)

\[ k_{\text{at}} \delta_0 = \frac{1}{a} + \frac{1}{2} r_0 k^2 + \cdots \]

effective range approximation
proton-proton scattering

\[ \frac{d\sigma}{d\Omega} = \left( \frac{e^2}{4\pi\alpha} \right)^2 \frac{1}{4\lambda^2} \left\{ \frac{1}{\sin^2 \theta/2} + \frac{1}{\cos^2 \theta/2} - \frac{\cos \left( \eta \sin \frac{\theta}{2} \right)}{\sin \frac{\theta}{2} \cos \frac{\theta}{2}} \right\} : \text{Coulomb scattering} \]

\( \theta \rightarrow \theta' \) : \text{interference (purely quantum)}

- Fermion, odd parity, Pauli exclusion principle, \( S=0 \), singlet

\[ -\frac{2}{\eta^2} \sin^2 \theta - \left( \cos \left( \theta + \eta \sin \frac{\theta}{2} \right) \right) + \frac{\cos \left( \theta + \eta \sin \frac{\theta}{2} \right)}{\cos^2 \theta/2} \]

: interference between Coulomb and nuclear scattering

\[ + \frac{4}{\eta^2} \sin^2 \theta \] : nuclear scattering

- \( T \): kinetic energy of the incident proton in lab frame

\[ \eta = \frac{\alpha}{\beta}, \quad \alpha = \frac{e^2}{4\pi\alpha_0\hbar c} = \frac{1}{137} \quad \text{fine structure const}, \quad \beta = \frac{\nu}{c} \]

- \( Q \rightarrow 0 \): All Coulomb terms goes 0, leaving only \( \sin^2 \theta \) term.

- Experimenter: \( \frac{d\sigma}{d\Omega} \) vs \( \theta \)

\[ \begin{array}{c}
\frac{d\sigma}{d\Omega} \\
\theta \end{array} \]

- total scattering length \( a = -7.82 \text{ fm} \), \( R_s = 2.79 \text{ fm} \)

- nuclear scattering length \( a = -17.1 \text{ fm} \), \( R_0 = 2.84 \text{ fm} \)

\( Q < 0 \), no pp bound state
Neutron - neutron scattering

- Experimentally difficult
  
  only by nuclear reaction:

  \[
  \begin{align*}
  \pi^- + ^2H &\rightarrow 2n + \gamma \\
  n + ^3H &\rightarrow 2n + P \\
  ^3He + ^2H &\rightarrow ^3H + 2P \\
  ^3H + ^2H &\rightarrow ^3He + 2n
  \end{align*}
  \]

- \[ a = -16.6 \text{ fm} < 0, \text{ no bound state} \]
  
  \[ r_0 = 2.66 \text{ fm} \]

- It's tempting to explain the non-existence of PP bound state by Coulomb repulsion. But it's due to nuclear force.
  
  See no PP bound state.

- Different from np scattering length -17 vs -23

  sensitive to wave function near \( R \)

  -23 -17

- Exchange force model

  vs. Covalent bond between two atoms, share electrons.

  Interaction with 3rd atom is weak

  - meson (boson)
Nuclear Shell Model

- Many-body problem
- Many-body interaction
- Atomic orbital

![Graph 1: Atomic orbital vs nuclear self-field](image)

- 2n separation energy $S_{2n}$
- 2p $S_{2p}$

![Graph 2: Stability Chart](image)
potential

- infinite square well (nucleon cannot get outside)
  \[ n \text{ can be larger than } \lambda \]

- 3D harmonic potential (no separation energy, long range)
  \[ V(r) = -\frac{V_0}{1 + \exp \frac{r - R}{\alpha}} \]

- \( Q = 0.52a \) fm
- \( V_0 \sim 50 \text{ MeV} \)
- \( R = 1.25 \, A^{1/3} \)

- spin-orbit coupling vs atomic model
- split of \( L \) degeneracies
- fine structure

- n.p. fill up the shell independently

- Nuclear magic number
  \[ 2, 8, 20, 28, 50, 82, 126 \]

- Valence nucleons \( \Rightarrow \) determine nuclear properties (spin, parity, multipole, quadrupole...)

- Excited states \( \Rightarrow \) radioactive decay, nuclear reaction

- Dynamics - nuclear vibration - nuclear rotation
Interaction of Neutron with Matter
- depends on energy of neutrons (thermal, epithermal, alpha, resonance, fast)
- depends on target nucleus

Classifications
- elastic scattering \( (n,n) \) \( Q=0 \) potential scattering
- resonance scattering: formation and decay of a compound nucleus
- radiative capture \( (n,r) \)
- inelastic scattering \( (n,n') \): excitation of nuclear levels \( Q\neq 0 \)
- particle emission \((n,p), (n,\alpha), \ldots\)
- fission \( (n,f) \)

Kinematics of two-body interaction, \( Q \)-equation
- Assume \( E_2=0 \). \( (E_1 \rightarrow E_2) \), applies to slowing down vs. thermalization

\[ \begin{align*}
E_{\text{kinetic energy}} & = \int \text{rest mass} \\
\text{Energy conservation} & \quad (E_1 + m_1c^2) + (E_2 + m_2c^2) = (E_3 + m_3c^2) + (E_4 + m_4c^2) \\
\text{Momentum conservation} & \quad \vec{P}_1 + \vec{P}_2 = \vec{P}_3 + \vec{P}_4 \\
& \quad |\vec{P}_1 - \vec{P}_3|^2 = |\vec{P}_4|^2 \\
& \quad P_1^2 + P_2^2 - 2P_1P_2\cos\theta = P_4^2 \\
\end{align*} \]

\[ Q = (m_1 + m_2 - m_3 - m_4)c^2 = E_3 + E_4 - E_1 = E_3 - E_1 + \frac{P_4^2}{2m_4} = E_3 - E_1 + \frac{1}{2m_4}(P_1^2 + P_2^2 - 2P_1P_2\cos\theta) = E_3 - E_1 + \frac{1}{2m_4}(2m_1E_1 + 2m_2E_2 - 2\sqrt{2m_1E_1m_2E_2}\cos\theta) = E_3(1 + \frac{m_1}{m_1}) - E_1(1 - \frac{m_1}{m_2}) - \frac{2}{m_4}\sqrt{m_1m_2E_1E_2}\cos\theta \]

\( Q \)-equation
Given $E_1$, mass $Q$, solve $E_3$ in terms of $Q$, quadratic in $\sqrt{E_3}$

- Elastic scattering, $Q=0$, in Laboratory Coordinate System (L)
  - Primary mechanism by which neutrons lose energy in a reactor
  $m_1=m_3=m$, $m_2=m_4=Am$

Q equation becomes

$$E_3 \left(1 + \frac{1}{A}\right) - E_1 \left(1 - \frac{1}{A}\right) + \frac{2}{A} \sqrt{E_1 E_3 \cos \theta} = 0$$

$$E_3 - \frac{2}{A+1} \sqrt{E_1 \cos \theta} \sqrt{E_3} - \frac{A-1}{A+1} E_1 = 0$$

$$\sqrt{E_3} = \frac{1}{2} \left( \frac{2}{A+1} \sqrt{E_1 \cos \theta} \sqrt{E_3} + \sqrt{\left(\frac{2}{A+1}\right)^2 E_1 \cos \theta + 4 \left(\frac{A-1}{A+1}\right)^2 E_1} \right)$$

- Only keep the + solution, the - solution is negative

- $\theta=0 \Rightarrow E_3 = E_1$ Forward scattering, no interaction
- $\theta=\pi \Rightarrow E_3 = \left(\frac{A-1}{A+1}\right)^2 E_1 = \frac{1}{A} E_1$ Backward scattering

- Elastic scattering in Center of Mass Coordinate System (CM)

L: $O \rightarrow v_1 \rightarrow v_0$

CM: $O \rightarrow v'_1 \rightarrow v'_2$

in L, center of mass velocity $v_0 = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{1}{A+1} v_1$

$$v'_1 = v_1 - v_0 = \frac{A}{A+1} v_1 , \quad v'_2 = 0 - v_0 = -\frac{1}{A+1} v_1$$

$$E_3 = \frac{1}{2} m v'_3^2 = \frac{1}{2} m \left( v'_1^2 + v'_2^2 + v'_0^2 \right) = \frac{1}{2} m \left( v'_1^2 + v'_2^2 + 2 v'_1 v'_0 \cos \theta_c \right)$$

$$= \frac{1}{2} m \left( \left(\frac{A}{A+1}\right)^2 v_1^2 + \left(\frac{A}{A+1}\right)^2 v_0^2 + 2 \frac{A}{A+1} v_1 \frac{1}{A+1} v_1 \cos \theta_c \right) = \frac{1}{2} E_1 \left[ (1+\alpha) + (1-\alpha) \cos \theta_c \right]$$
Energy distribution \( F(E \to E') \) scattering frequency, conditional probability:

- **s-wave angular distribution** (in cm):
  \[
P(\theta_c) d\theta_c = \int_0^{2\pi} P(\theta_c) \sin \theta_c d\theta_c d\phi = \frac{1}{2} \sin \theta_c d\theta_c \Rightarrow P(\theta_c) = \frac{1}{2} \sin \theta_c
  \]

- \( E_1 = E, \quad E_3 = E' \)

\[
F(E \to E') dE' = P(\theta_c) d\theta_c
\]

\[
F(E \to E') = P(\theta_c) \left| \frac{d\theta_c}{dE'} \right|
\]

\[
\frac{dE'}{d\theta_c} = \frac{1}{2} E (1-\alpha)(-\sin \theta_c) \Rightarrow \frac{d\theta_c}{dE'} = \frac{2}{E (1-\alpha) \sin \theta_c}
\]

\[
F(E \to E') = \frac{1}{2} \sin \theta_c \cdot \frac{2}{E (1-\alpha) \sin \theta_c} = \frac{1}{E (1-\alpha)} \quad \forall E \leq E' \leq E
\]

\[
F(E \to E') = \begin{cases} 
\frac{1}{E (1-\alpha)} & \text{if } E \leq E' \leq E \\
0 & \text{otherwise}
\end{cases}
\]

![Graph](image)

Useful to study moderation

**Ex.** - hydrogen \( A=1, \alpha=0 \), full range \((0, E)\)

- \( A \gg 1, \alpha \gg 1 \), narrow region \((dE, E)\)

**Energy differential cross section**

\[
\frac{d\sigma_s}{dE'} = \sigma_s(E) F(E \to E')
\]

\[
\sigma_s(E) = \int \frac{d\sigma_s}{dE'} dE'
\]

\[
N \sigma_s(E) F(E \to E') = \Sigma_s(E \to E')
\]

scattering kernel in

neutron transport equation
Average energy loss:
$$\langle \Delta E \rangle = \int_0^E (E-E') F(E \rightarrow E') dE' = \int_0^E \frac{E-E'}{E(1-\alpha)} dE' = \frac{E}{2} (1-\alpha)$$

- hydrogen, $\Delta E = \frac{E}{2}$
- $A \gg 1$, $\Delta E = \frac{2E}{A}$

Angular distribution:

- in CM
  $$\frac{d\sigma}{d\Omega} = \sigma_s(\theta_c) = \sigma_s(E) \frac{1}{4\pi}$$

- in L
  $$\sigma_s(\theta) d\Omega = \sigma_s(\theta_c) d\Omega_c$$
  $$\sigma_s(\theta) d\Omega = \sigma_s(\theta_c) \frac{\sin \theta_c d\theta_c}{\sin \theta d\theta} = \sigma_s(\theta_c) \frac{d\cos \theta_c}{d\cos \theta}$$
  $$\frac{d\cos \theta_c}{d\cos \theta} = \frac{A}{\sqrt{A^2 + 1 + 2 A \cos \theta_c}} - \frac{1 + A \cos \theta_c}{2} \cdot 2A = \frac{A^2 (1 + \cos \theta_c)}{(A^2 + 1 + 2 A \cos \theta_c)^{3/2}}$$

- where $\gamma = \frac{A}{1}$

    $$\sigma_s(\theta) = \frac{\sigma_s(E)}{4\pi} \frac{(A^2 + 2 A \cos \theta_c + 1)^{3/2}}{1 + A \cos \theta_c}$$

Peaked in the forward direction because $0 < \theta < \theta_c$.

Average:
$$\mu = \cos \theta$$

$$\langle \mu \rangle = \frac{\int \mu \sigma_s(\mu) d\mu}{\int \sigma_s(\mu) d\mu} \approx \frac{\int \frac{1}{\sigma_s(\mu)} \sigma_s(\mu) d\mu}{\int \frac{1}{\sigma_s(\mu)} d\mu} = \frac{2}{3A}$$

$\langle \mu \rangle > 0$ peaked in the forward direction.

$A \gg 1$, $\langle \mu \rangle \approx 0$. 
• Three assumptions

1) elastic scattering. \( Q=0 \), no reaction \( \sim 5-10 \) b, (20 b for H)

inelastic scattering \( \sim 1 \) b

2) target nucleus at rest

\( A=1 \), hydrogen

\( E > kT \), \( 0.1 \text{eV} < E < 0.3 \text{MeV} \)

• Energy dependence of scattering cross section \( \sigma_s(E) \)

\( \sigma_{\text{free}} = \sigma_{\text{bound}} \left( \frac{A}{A+1} \right)^2 \)

\( \sigma_s \propto m^2 \left( \frac{m}{m+n} \right)^2 \left( \frac{A}{A+1} \right)^2 \)

\( b = \frac{A+1}{A} \)

\( b \) bound scattering length

\( a \) free scattering length

\( \Theta_t \) intramolecular motions

\( \Theta_f \) free vibrations

\( 2(\Theta_t(H) + \Theta_f(\text{H})) \)
3) Scattering is isotropic (s-wave) \( E < 10 \text{ keV} \)

\[ F(E \rightarrow E') \]

- Backward scattering
- Forward scattering, \( p\)-wave...

\( dE \rightarrow E \rightarrow E' \)
Interaction of γ-ray with matter

- γ: λ < 0.01 nm, \( E > 120 \text{ keV} \) (0.1 - 10 MeV)
- primarily produced by nuclear transitions
- neutron sees nuclei (nuclear force)
- \( \gamma \): electrons, E field
- charged particle: both nuclei and electrons (Coulomb force)

• Attenuation

\[ I(x) = I_0 e^{-\mu x} \]

\( I_0 \): attenuation coefficient,
\( I(x) \): probability per unit path of interaction
\( vs \) macroscopic cross section \( \Sigma = N_0 \sigma \)

\( \frac{I}{I_0} \): transmission coefficient

\( \mu \): Compton + photoelectric + pair production

Interaction with electric field

- Scattering
- Absorption

PS: If \( E \) is small,

Compton \( \rightarrow \) Thomson

inelastic scattering \( \rightarrow \) elastic scattering

relativistic \( \rightarrow \) non-relativistic
\[ m = \gamma m_0 = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \]
\[ \hbar k = \hbar k' \cos \theta + \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} v \sin \phi \]
\[ \hbar k' \sin \phi = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} v \sin \phi \]
\[ \hbar k^2 + h^2 k'^2 - 2 \hbar k k' \cos \theta = \frac{m_0^2 v^2}{1 - \frac{v^2}{c^2}} \quad (\sim \mathcal{E}) \]
\[ (\hbar k - \hbar k' \cos \theta)^2 + (\hbar k' \sin \phi)^2 = \frac{m_0^2 v^2}{1 - \frac{v^2}{c^2}} \quad (= \mathcal{P}^2) \]
\[ \frac{\hbar^2 k^2 + \hbar^2 k'^2}{2} = \frac{m_0^2 v^2}{1 - \frac{v^2}{c^2}} \quad (\sim \mathcal{E}) \]
\[ \Delta \lambda = \lambda' - \lambda = 2\pi \left( \frac{1}{k} - \frac{1}{k'} \right) = \frac{2\pi \hbar}{m_0 c} \left( 1 - \cos \theta \right) \quad \text{Compton shift} \]
\[ \lambda_c = \frac{2\pi \hbar}{m_0 c} = 2.426 \times 10^{-12} \text{m} \quad \text{Compton wavelength} \]
- kinetic energy of the recoil electron (Compton electron)

\[ T = \frac{1}{2} m c^2 - \frac{1}{2} \frac{2 \hbar c}{\lambda} = \frac{1}{2} \frac{2 \hbar c}{\lambda + \lambda c (1 - \cos \theta)} = \frac{1}{2} \frac{2 \hbar c}{\lambda c} \left[ 1 - \frac{2 \hbar c}{\lambda c (1 - \cos \theta)} \right] \]

\[ \alpha = \frac{\hbar \nu}{m_0 c^2}, \quad \text{relative energy ratio} \]

\[ \frac{W'}{W} = \frac{1}{1 + \frac{\alpha (1 - \cos \theta)}{\sin \theta}} \]

Ex. 1: \[ \frac{W'}{W} = \frac{1}{1 + \frac{\alpha (1 - \cos \theta)}{\sin \theta}} \]

- High energy \( \nu \) suffer large energy loss.

2. When \( \alpha \gg 1 \), \( W' \approx \frac{W}{\alpha (1 - \cos \theta)} = \frac{m_0 c^2}{\hbar \nu (1 - \cos \theta)} \) only depends on \( \theta \).

\( \Theta = \pi \), backward scattering, \( \hbar \nu = \frac{1}{2} m_0 c^2 \)

\( \Theta = \frac{\alpha}{2} \)

\( \Theta = 0 \), \( W' = \frac{W}{1 + \frac{\alpha \nu}{m_0 c^2}} = W \), no interaction, forward scattering

- Klein-Nishina cross section, angular distribution of photon

1. \( \frac{d \sigma}{d \Omega} = \frac{e^2}{4 \pi} \left( \frac{W'}{W} \right)^2 \left( \frac{W}{W'} + \frac{W'}{W} - 2 + 4 \cos^2 \Theta \right) \) polarized photon

\( \Theta \): angle between the polarization of the incident and scattered photon

\( r_e = \frac{e^2}{m c^2} \): classical radius of the electron

2. unpolarized photon

\( \frac{d \sigma}{d \Omega} = \frac{e^2}{2} \left( \frac{W}{W'} \right)^2 \left( \frac{W}{W'} + \frac{W'}{W} - \sin^2 \Theta \right) \)

\[ = \frac{e^2}{2} \frac{1 + \cos^2 \Theta}{(1 + \alpha (1 - \cos \Theta)) \left[ 1 + \frac{\alpha^2 (1 - \cos \Theta)^2}{(1 + \cos \Theta)(1 + \alpha (1 - \cos \Theta))} \right]} \]
Energy distribution

\[
\frac{d\sigma}{d\theta} = \int_0^{2\pi} \frac{d\sigma}{d\Omega} \sin \theta \, d\phi = \frac{d\sigma}{d\Omega} \cdot 2\pi \sin \theta
\]

\[
\frac{d\sigma}{d\omega} = \frac{d\sigma}{d\Omega} \frac{d\phi}{d\omega} \quad \text{Compton photon}
\]

\[
\frac{d\sigma}{d\tau} = \frac{d\sigma}{d\Omega} \left| \frac{d\phi}{d\tau} \right| \quad \text{Compton electron}
\]

\[
T(\omega, \theta) = T(\omega, \phi)
\]
Photoelectric effect

\[ T \approx h\nu - Be \quad \text{\(w > w_c\)} \]

1) Atomic excitation (not nuclear excitation)

2) Excited atom \( \rightarrow \) de-excite by emitting x-rays or Auger electrons

3) Not free electron; it cannot be absorbed by a free electron because momentum and energy conservation cannot be satisfied simultaneously.

- K shell (innermost) Al: 23 keV
  Cu: 10 keV
  Pb: 160 keV

- \( k \) edge absorption

\[ \int k^2 \, dt = P + P_a \]

\[ k\omega = T + Ta + Be \]

\( Ta \sim T \frac{Me}{M} \), usually can be ignored.

Cross section: first-order perturbation theory

\[ \frac{d\sigma}{dz} = 4 \sqrt{\frac{137 \gamma}{Z^5}} \frac{\sin^2 \theta \cos^2 \theta}{(1 - \frac{1}{Z})^4} \]

\[ \sigma = \int d\sigma \, dz = 4 \sqrt{\frac{137 \gamma}{Z^5}} \frac{(M \omega^2)^{\gamma}}{(h\omega)^2} \]

In practice

\[ \sigma \propto \frac{Z^n}{(h\omega)^3} \]

Where

\[ n = 4 - 4.6 \]

\( h\omega = 0.1 - 3 \) MeV

\[ Z = 1 \]

\[ Z = 82 \]

\[ \frac{\sigma}{Z^2} \frac{d\sigma}{dz} \]

(\( \frac{10^{20}}{cm^2 \cdot ster} \))
Pair production

- $E_x > 2m_e c^2 = 1.022 MeV$

\[ \gamma \rightarrow \beta^+ + \beta^- \]

$\beta^+ + \beta^- \rightarrow 2\gamma$, $E_x = 0.511 MeV$ annihilation. PET

- Intimately related to Bremsstrahlung radiation

- Energy distribution

\[ \frac{d\sigma}{dT^+} = 4\pi \frac{2}{(\hbar\omega)^3} \frac{T_+^2 + T_-^2 - \frac{2}{3}T_+T_-}{\ln \left( \frac{2T_+T_-}{\hbar\omega m_e c^2} \right)} \]

\[ = \frac{\sigma_0 Z^2}{\hbar\omega - 2m_e c^2} \cdot P \left( \frac{\hbar\omega}{2m_e c^2} \right) \]

- Slightly more energy for positrons because of Coulomb repulsion from the nucleus
\[ \sigma_k = \left( \frac{d \sigma_k}{dT} \right) = 0.5 Z^2 \left[ \frac{28}{9} \ln \left( \frac{2 \hbar w}{m_0 e^2} \right) - \frac{218}{27} \right] \text{ no screening} \]

\[ \sigma_k = 0.5 Z^2 \left[ \frac{28}{9} \ln \left( \frac{183}{Z^2} \right) - \frac{2}{27} \right] \text{ complete screening} \]
\[ \mu = \mu_e + \mu_r + \mu_k \]

\( \mu_p \): mass attenuation coefficient - more or less independent of the density and the physical state of matter

\[ \frac{\mu_e}{P} = \frac{N_o}{A} Z \sigma_e, \quad \sigma_e \sim \frac{1}{n \hbar^2} \text{ per electron} \]

\[ \frac{\mu_r}{P} = \frac{N_o}{A} \sigma_r, \quad \sigma_r \sim \frac{Z^2}{(\hbar \gamma)^2} \text{ per atom} \]

\[ \frac{\mu_k}{P} = \frac{N_o}{A} \sigma_k, \quad \sigma_k \sim Z^2 \ln \left( \frac{2h \nu}{m_0 c^2} \right) \text{ per atom} \]
NaI scintillation detector  low cost
    low energy resolution

- Pulse-height spectrum

  Photon peak $E_0$

  Compton edge $E_0 - \frac{1}{2}m_e c^2$

  Escape peak 1 $P_1 = E_0 - m_e c^2$

  Escape peak 2 $P_2 = E_0 - 2m_e c^2$

  Backscattering Compton scattering at $\theta = \pi$ $\frac{1}{2}m_e c^2$

  Annihilation photon $m_e c^2$

- CZT semiconductor detector
1. Interaction of charged particle with matter
   - Heavy: p, α, ...
   - Light: electron, positron
   - Interaction: Coulomb

2. Types of interaction
   1. Inelastic collision with the electrons (dominant)
      ⇒ cause excitations of atomic electrons, or ionization
   2. Inelastic collision with the nuclei (very high energy) (>10^2 MeV)
      ⇒ cause excitation of nuclei,
      or bremsstrahlung of the charged particle
   3. Elastic scattering with the nuclei (Rutherford)
      ⇒ The particle loses energy only through the recoil of the nuclei.
   4. Elastic collision with the electrons
      ⇒ only significant for low-energy electrons
Stopping power \[ -\frac{dT}{dx} \]
- the loss of kinetic energy per unit distance
- depends on \( v, Z, e \)
  \( Z, e, n \equiv \text{ionization potential} \)
- Bethe formula (crude derivation)

\[ \int F_x dx = 0 \]

\[ Pe = \int F_y dy = \frac{1}{4\pi\varepsilon_0} \int \frac{Ze^2}{x^2 + b^2} \frac{b}{v^2} \frac{dx}{v} = \frac{1}{4\pi\varepsilon_0} \frac{2Ze^2}{v_0} \]

\[ \frac{Pe}{2me^2} = (\frac{e^2}{4\pi\varepsilon_0}) \frac{2Ze^2}{me^2} \]

\[ -\frac{dT}{dx} = \int nZ \frac{Ze^2}{2me} \frac{dx}{v^2} = \left( \frac{e^2}{4\pi\varepsilon_0} \right) \frac{4\pi Z^2 n Z}{me^2} \ln \frac{\text{max}}{\text{min}} \]

in reality, no free electrons \( \text{max} = \frac{h\nu}{I} \)

\[ I \approx kZ, \quad k = 19 \text{eV for } H, \quad 10 \text{eV for } Pb \]

\( \text{min} = \frac{h}{me^2} \) de Broglie wavelength

\[ -\frac{dT}{dx} = \frac{4\pi Z^2 n Z}{me^2} \left( \frac{e^2}{4\pi\varepsilon_0} \right) \ln \frac{2me^2}{I} \]

non-relativistic

\[ \ln \left( \frac{2me^2}{I} \right) - \ln (1 - \beta^2) = \beta^2 \]

relativistic

\[ \approx Z, \quad \frac{1}{v^2} \]

\[ n Z = p \frac{h}{A} \]

\[ Z \approx \text{const} \]

\[ \Rightarrow -\frac{1}{x} \frac{dT}{dx} \]
- primarily inelastic collision with electrons (electronic stopping power)

\[ \frac{d\varphi}{dx} \cdot \frac{Z}{M_{\text{me}}} \quad Z < 10^3, \quad \frac{M}{m_{\text{e}}} > 2 \times 10^5 \]

(nuclear stopping power) (much less)

\[ \frac{d\varphi}{dx} \] is independent of the mass of the charged particle (only \( Z \))

Experiment

![Graphs showing the relationship between energy transfer and energy loss per unit path length for different particle energies.]

Schematic

![Diagram illustrating the process of electron capture and related terms.]

- Bohr classical result, similar

\[ \tilde{i} = \frac{1}{W} (-\frac{d\varphi}{dx}) \]

number of ion pairs per unit distance (specific ionization)
Radiation loss (Bremsstrahlung)

Larmor formula: $P = \frac{\mu_0 q^2 a^2}{6\pi c}$

$-\frac{dT}{d\alpha} \propto \left(\frac{Z}{m}\right)^2 = \left(\frac{Z}{m}\right)^2$ important for high $Z$ material $e$ and $\beta^+$

![Graph showing radiation loss vs. $T$ and $\log$ scale for ionization loss and radiation loss]

Cerenkov radiation

$\beta > \frac{c}{n}$ speed of light in the media
- Range

\[ R = \int_0^R \frac{d\chi}{dx} \frac{d\chi}{dT} dT = \int_0^T \frac{dT}{\frac{dT}{dx}} dT \]

Using Bethe formula: \( R \propto \int_0^T TdT = T_0^2 \) \hspace{1cm} (Range - Energy relation)

valid at low energy, \( \frac{dT}{dx} = \text{const.} \)

\[ R \propto \int_0^T dT = T_0 \]

- Bragg curve

\[ \alpha \approx 5.49 \text{MeV in Air} \]

- 1MeV \( \gamma \) in Al. half thickness = 4.2 cm

- \( e \) \( R = 1.8 \text{ mm} \)

- \( \alpha \) \( R = 3 \mu \text{m} \)